

Investigation of evolution of orbits similar to that of (4179) Toutatis

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Abstract

This project compared the orbital evolution of asteroid (4179) Toutatis to that of several fictitious asteroids in similar orbits. The nominal orbit of Toutatis along with orbits with varying initial values of inclination and eccentricity were numerically integrated for a period of 10,000 years. A limited run of 1,500 years using the nominal orbit for Toutatis but with a different timestep confirmed the effect of its highly chaotic behavior on attempts to numerically integrate this orbit. Runs with various inclinations showed a variety of chaotic behaviors resulting from the frequent close encounters with the inner planets. Dependence of the frequency of encounters on the orbital inclination relative to the inner planets was observed. Encounters usually but not always disrupted the 3:1 mean motion resonance with Jupiter. A test using the orbital elements of Toutatis but an eccentricity of 0.0 showed periodic behavior with the eccentricity increasing to 0.12 in a period of 10,000 years.

1 Introduction

An asteroid is in a mean motion resonance with a planet when its orbital period is near a ratio $p : q$ of the planet's period, with p and q both integers. When this is the case, conjunctions of similar geometry between the asteroid and planet recur at periods of p times the orbital period of the asteroid (or q times the period of the planet), and the effect of perturbations can accumulate significantly before the geometry shifts to have a different effect. Kirkwood first noted the gaps in the asteroid belt for values of semi-major axis corresponding to such resonances with Jupiter. Wisdom (1982, 1983) studied fictitious asteroids near the 3:1 resonance, corresponding to a semi-major axis of 2.501 AU. In his tests, asteroids near the resonance with eccentricities less than 0.1 would undergo small variations in eccentricity for timescales of 10^5 years, then suddenly undergo chaotic eccentricity increases to over 0.3. If the resultant orbit is Mars-crossing, the asteroid can become Earth-crossing on timescales of 10^6 years. This has been discussed as a plausible source for Earth-crossing asteroids as well as for meteorites. At certain commensurabilities, such as

3:2, 4:3, and 1:1, concentrations of asteroids are observed. These values of semi-major axis have been shown to be stable for small eccentricities by Yoshikawa (1989). On shorter timescales, mean motion resonances can be stable by protecting the asteroid from close approaches: for example, a resonance state with Jupiter where the recurring geometry puts the asteroid at aphelion when Jupiter is on the opposite side of the Sun can protect the asteroid from the closest possible approaches to Jupiter and hence from the largest perturbing effects.

Secular resonances occur when an asteroid's rate of precession of longitude of perihelion (or longitude of ascending node) is near a ratio $p : q$ of that of a major planet. The location of such resonances with Jupiter and Saturn have been determined analytically according to dependence on a , e , and i . Gaps in the distribution of asteroids are also observed at these locations, and dynamical studies have shown that these resonances too can produce chaotic orbital evolution. For low values of inclination, the g_5 and g_6 secular resonances (which correspond to rates of precession of longitude of perihelion equal to those of Jupiter and Saturn, respectively) correspond to a semi-major axis of about 2.5 AU (Knezevic *et al.*, 1991).

For main-belt asteroids the resonances associated with Jupiter and to a lesser extent Saturn are dominant. Asteroids which approach the inner planets may undergo mean motion resonances with them. A number of Earth-crossing asteroids have been identified as being in such resonances with the Earth or Venus where the resonance serves to protect the asteroids from close approaches as discussed earlier. Such conditions are not stable over long periods of time, and dynamical studies of such objects show that they frequently enter and exit such resonances (Ip and Mehra, 1973; Milani *et al.*, 1989). Encounters with the inner planets can produce more chaotic orbital behavior—as in the case of Toutatis.

2 Toutatis

The Earth-crossing asteroid 1989 AC was discovered in 1989 and subsequently linked with 1934 CT before receiving the permanent designation (4179) Toutatis. Radar observations in 1993 revealed that Toutatis is a contact binary with a total length of at least 3.5 km (Ostro *et al.*, 1993). With an orbital period of 3.976 years it is in a 3:1 mean motion resonance with Jupiter and a weak 1:4 resonance with the Earth. Using observations from 1934 to 1993 Whipple and Shelus (1993b) have computed the orbital elements given in table 1.

The orbit of Toutatis has an inclination to the ecliptic of only 0.4616° , which makes it the only numbered Mars-crossing asteroid with an inclination below one degree. This permits frequent close approaches to the inner planets. With this low inclination, the maximum separation between the orbits of Toutatis and the Earth (as opposed to the bodies themselves) is 0.008 AU or less, depending on the position of the ascending node. The dynamical study of Whipple and Shelus shows that these frequent encounters produce a very high degree of chaos in the orbital behavior of Toutatis, several orders of magnitude higher than the degree of chaos associated with the 3:1 resonance with Jupiter. Another factor contributing

to frequent encounters (although less important than low inclination) is proximity of the perihelion to that of an inner planet, since near perihelion a greater segment of the asteroid's orbit is near a specific heliocentric distance. (Table 2 lists other asteroids near the 3:1 resonance with high eccentricities.)

3 Objectives

Given the chaotic nature of Toutatis' orbit produced by the perturbations of the inner planets, the goal of this project was to examine similar orbits to test the dependence of this behavior on the specific characteristics of Toutatis' orbit. The primary focus was its uncommonly low inclination to the ecliptic for an Earth-crosser. Beginning with the nominal orbital elements for Toutatis, the inclination and eccentricity were varied with several test orbits numerically integrated for a period of 10,000 years.

4 Procedure

The integrator program for the tests was provided by A. Whipple. This model uses a standard timestep except during close approaches to the planets, when the timestep is reduced to maintain accuracy. Masses and initial elements for the major planets are from the DE200 ephemeris and are given in table 3. The Earth and Moon are merged as a single mass at the Earth-Moon barycenter and the test object's mass is neglected. Output consists of the Cartesian state vectors for the planets and test object at each standard timestep and at specified intervals during close approaches. From this output a companion program calculates the orbital elements referred to the invariable plane of the solar system. In addition, data on close approaches is recorded for those where the maximum acceleration produced by the planet exceeds a specified fraction (in these tests, 0.2%) of the solar acceleration.

A program written by the author used the state vectors file to determine the inclination of the asteroid's orbit with respect to each of the planet's orbits. For each relevant body a vector perpendicular to both the position and velocity vectors was obtained, this being the angular momentum vector. Then the angle between the angular momentum vectors for the asteroid and the planet in question was calculated, this being the relative orbital inclination.

The integration program also outputs calculations for the Lyapunov characteristic exponent (LCE) as a measure of the degree of chaos in the orbital behavior. An auxiliary orbit is simultaneously integrated, initially differing in phase space by the vector quantity \vec{d}_0 . At regular intervals of length t the quantity

$$k_n = \frac{1}{nt} \sum_{i=1}^n \ln \left(\frac{\vec{d}_i}{\vec{d}_0} \right)$$

is calculated, where n is the number of timesteps and \vec{d}_i the displacement after the i th timestep. As the time goes to infinity, k_n approaches the

maximum LCE, which is zero for non-chaotic quasi-periodic behavior and is higher for more chaotic behavior. While the tests in this investigation are generally not long enough for k_n to converge, this still gives a relative measure of the degree of chaos.

The programs were run on the UT astronomy department ASTRO computer. The standard run period was 10,000 years. A standard timestep of 920 days was used (this is approximately the orbital period of Toutatis divided by π , an arbitrary figure avoiding commensurabilities with its period). Each test generally required about 60 hours of computer time, or from three to six days real time depending on the level of ASTRO usage. The test cases are listed in table 4 with starting orbital elements.

5 Tests

5.1 Test 1

The control run used the nominal orbital elements for (4179) Toutatis and the standard 920-day step. Evolution of orbital elements and close approach data are plotted in figure 1. The test object (which can be considered to represent the real object Toutatis for only a limited period of time) makes repeated close approaches to the Earth during the first 1,500 years (the total number of encounters is given in table 4). Fig. 1 includes graphs of the critical arguments for the resonances with Jupiter and the Earth— $\sigma_{3:1}$ and $\sigma_{1:4}$, respectively. The critical argument is defined as

$$\sigma_{p:q} = q\lambda - p\lambda_p + (p - q)\omega,$$

where p and q are integers, λ and λ_p are the mean longitudes of the asteroid and planet, respectively, and ω is the longitude of perihelion of the asteroid. Libration of the critical argument about a value of 180° indicates a resonance state protecting against close approaches. The graph of $\sigma_{1:4}$ indicates resonance interactions with the Earth (although not a sustained resonance). The longest of these lasts from $t = 100$ to $t = 300$ years, "protecting" the object from close approaches to the Earth in this period. A persistent decrease in eccentricity ends approaches to the Earth, as by about $t = 2,000$ years the orbit is no longer Earth-crossing. The graphs of the orbital elements show that a corresponding regularity to the cyclic behavior sets in. In this test the object was never in resonance with Jupiter. In the first 1,000 years encounters with the Earth disrupt such a resonance. Once these cease, however, the object is left with a semimajor axis librating from about 2.54 to 2.58 AU; probably this is too far from the 3:1 resonance (at 2.501 AU) for this to take hold. Fig. 2 plots a vs. $\sigma_{3:1}$, which would depict an elliptical figure if a resonance occurred (as in fig. 7 for the following test). Fig. 3 plots $\log k_n$ vs. $\log t$, which represents the degree of chaos as discussed previously.

The graphs of perihelion and aphelion distances (fig. 4) contrast the effects of Jupiter and the inner planets. Close encounters with the inner planets have a larger (immediate) chaotic effect on aphelion distance than on perihelion distance, as expected: such encounters occur when the object is near perihelion, immediately causing little displacement but a

velocity change that alters the aphelion distance half an orbit later. By contrast, the regular cyclic behavior observed in perihelion distance but more subdued (and initially below the level of chaos) in aphelion distance is primarily due to the perturbations by Jupiter, which are most pronounced at aphelion.

Fig. 5 shows the evolution of $p = \sin(i) * \sin(\Omega)$ and $q = \sin(i) * \cos(\Omega)$, where i is the inclination to the invariable plane and Ω is the longitude of the ascending node. This corresponds to the projection of the object's angular momentum vector (rotated 90°) onto the invariable plane. The figure shows a characteristic periodic behavior with a period of about 7,000 years. The irregularities near $t = 0$ represent the chaotic effect of close encounters with the Earth. This periodicity corresponds to the cycles seen in the fig. 1 plot of the inclination of the object's orbit to the invariable plane and the orbits of Venus, the Earth, and Mars. Each planet's orbit is simultaneously precessing about the invariable plane due to their mutual perturbations; corresponding plots of p and q show somewhat more circular or elliptical cycles about the invariable plane with periods of several $\times 10^4$ years.

As the object's orbit oscillates about the invariable plane, its inclination with respect to the orbits of each of the planets varies. The orientation of the angular momentum vector (and thus the track of p and q) is close to that of one of the planets when its orbital inclination to that planet's orbit is low. Thus, as its $p - q$ track oscillates it may pass near the corresponding track of one of the planets, which would correspond to an opportunity for close approaches if the orbits cross. The inclination of the Earth's orbit to the invariable plane is smaller than that for Mars and Venus, so its corresponding $p - q$ track about the invariable plane is smaller. This makes it more likely that the object's track will pass near the Earth's, *i.e.* more likely that a low inclination to the Earth's orbit will occur. The object's inclination to the Earth is less than one degree twice in the 10,000-year run, compared to once for Mars and never less than two degrees to Venus' orbit.

5.2 Test 2

An additional shorter (1,500 year) run using the nominal Toutatis elements was conducted using a time step of 460 days, or half the standard. The results differed unexpectedly from the control run. Examining the data on close encounters shows that the encounter with the Earth at $t = 486$ years is at a distance of 0.02202 AU versus 0.02204 AU for the control run. The next encounter (again with the Earth) at $t = 579$ years is at a distance of 0.01745 AU versus 0.01715 AU for the control. Listed close encounters diverge at this point; the next encounters are at $t = 745$ and 749 years (in a brief "kinematic" resonance connected to the 1:4 resonance with the Earth) versus one at $t = 741$ years in the control case. Subsequently the overall orbital evolution diverges, as seen in figure 6. In this run, resonance with Jupiter begins by $t = 1,000$ years and lasts to the end of the run; fig. 7 plotting a versus $\sigma_{3:1}$ shows resonance behavior. Brief resonance interactions with the Earth recur in this period. The object in this case does not avoid the resonance by being kicked to a higher

semi-major axis as in the control run.

This not only demonstrates the highly chaotic nature of Toutatis' orbit, it also confirms that it is sufficiently chaotic to restrict the validity of the numerical integration method in terms of evaluating the behavior of the real object, as observed by Whipple and Shelus (1993b). In their investigation very slight changes in the initial orbital elements resulted in completely different overall behavior within a few thousand years. In the test here the initial orbital elements were unchanged; only the time-step of the integration was cut in half. Previous authors have noted that numerical integration is only briefly applicable to the evolution of the real object in question, but for a longer term is informative about the behavior of a population of objects; Toutatis is an extreme case.

5.3 Test 3

Inclination was varied in the next series of tests. The initial inclination to the ecliptic for this run was 2.0° with the other orbital elements the same as the nominal elements for Toutatis. This object is in resonance with Jupiter throughout the run even though it still experiences frequent encounters with the Earth. As shown in fig. 8, repeated encounters with the Earth occur continuously—except from $t = 3,200$ to $t = 5,000$ years—along with a few encounters with Mars and Venus. The two encounters with Mars at $t = 6,200$ years and $t = 7,500$ years correspond to a low inclination relative to the orbit of Mars (see fig. 8). The object's orbit is not Venus crossing until the end of the run, when the relative inclination is higher; however, the perihelion is near the orbit of Venus at this time. Encounters with the Earth are frequent despite the higher inclination, although the closest encounter at $t = 7,160$ years corresponds closely to the lowest relative inclination during the run. The plot of $\log k_n$ —and hence the degree of chaos—in figure 10 begins to increase again around $t = 5,000$ years, when encounters with the Earth resume.

It is interesting that frequent encounters never disrupted the resonance with Jupiter, indicating that such a state is not uniquely dependent on an absence of chaotic encounters. For much of the run the resonance is relatively deep and perhaps less susceptible to disruption. An example is the encounter at $t = 7,160$ years when the object passes the Earth at only 1.5 times the distance to the Moon; this essentially kicks the object to the opposite point in the cycle in semi-major axis without otherwise disturbing the resonance. This is during the period of relatively deep resonance. Examination of the various encounters indicates that they occur at various points throughout the resonance cycle, ruling out a direct link to this in terms of disruptive effect.

Another interesting aspect of this run is the relatively chaotic track of the angular momentum vector (fig. 11). This track does not show the periodic behavior seen in the previous run. Moreover, this irregular behavior is simultaneous with the regular resonance behavior of the other elements.

5.4 Test 4

The next run used an initial orbital inclination of 5.0° with the other elements the same as the nominal elements. As seen in figs. 12-15, the object is in resonance with Jupiter for the first 2,700 years of the run, and only one close encounter with the Earth is recorded in this period. The disruption of the resonance is apparently not attributable to a particular close approach. Subsequently the object's shorter periodicity in semimajor axis is about a value of 2.43 to 2.46 AU, too far from the 3:1 resonance location for resonance to occur. In addition the orbital eccentricity progressively decreases until the object no longer crosses the Earth's orbit. The plot of the $p - q$ track (fig. 14) shows regular periodic behavior.

5.5 Test 5

In order to examine why the resonance in case 4 was disrupted, another test was conducted using the same initial elements but neglecting the effects of the inner planets on the test object. (The effects of the inner planets are still applied to Jupiter and Saturn so their behavior is unchanged.) This was in hopes of indicating whether the inner planets were responsible for the disruption of the 3:1 resonance in the previous test. Figs. 16-18 show the much more stable cycles that would occur without the perturbations of the inner planets. The 3:1 resonance did persist throughout the run; note again a longer term decrease (perhaps eventually cyclic) in eccentricity and the periodic behavior of the $p - q$ track. However, it is probably not directly indicated that the inner planets caused the disruption at $t = 2,700$ years in test 4. Superimposing the graphs of a and $\sigma_{3:1}$ from the two runs as in figure 20 shows a phase shift by the time of disruption, and thus conditions in the two tests do not correspond at $t = 2,700$ years. On the other hand, in the current test $\sigma_{3:1}$ librated about a value of 160° , from $5-20^\circ$ to about 300° . The plot suggests that in test 4 the object slipped out of resonance when the minima in $\sigma_{3:1}$ became too low. The resonance was relatively shallow and unstable, perhaps slightly more so when the cumulative perturbations of the inner planets were considered. Comparing the LCE calculations for the two runs (figs. 15 and 19) shows the much higher degree of chaos produced by inner planet encounters.

5.6 Test 6

The next run used initial elements with an inclination of 1.5° to the ecliptic and a longitude of ascending node of 217.0° (using the nominal Toutatis values for the other elements). The latter element is about 90° more than the nominal value for Toutatis; this was chosen to explore other areas of the $p - q$ phase space. Figs. 21 and 22 indicate that the object is in a relatively deep resonance with Jupiter throughout the run. The level of the resonance varies, primarily in two periods, first during the period of close approaches to the Earth from $t = 600$ years to $t = 3,800$ years. It also occurs between $t = 7,000$ and $t = 8,000$ years, when the only close approaches are two to Venus, and they would not appear to be entirely

responsible for the shift. Note that the variation in $\sigma_{3:1}$ is about a mean value of about 160° . As in test 3, the encounters with the Earth occur at points throughout the cycle in a and $\sigma_{3:1}$, indicating that this alone does not determine whether a close approach disrupts the resonance. No close approaches to the Earth occur after $t = 4,000$ years; beyond that the inclination to the Earth's orbit is over 4° and the eccentricity is increasing, reducing opportunities for Earth encounters. The eccentricity continues to increase until the object begins crossing the orbit of Venus, although the high inclination to the orbit of Venus probably explains the small number of encounters.

5.7 Test 7

This run again varied the inclination and longitude of ascending node (only), with initial values of 1.0° and 37.2323085° , respectively (the latter being 90° less than the nominal value for Toutatis). During the first 1,000 years the object shows some intermittent resonance interaction with Jupiter (fig. 24). There are then two very close encounters with the Earth, the second of which at $t = 1,067$ years is at only 0.6 times the distance to the Moon (this is the cause of the largest jump in the plot of the $p - q$ track). This apparently puts the object securely into resonance with Jupiter, lasting for about 800 years; this is a relatively deep resonance about a mean value of 200° for $\sigma_{3:1}$, and during this time it undergoes brief resonance interaction with the Earth several times.

The behavior over the next 3,500 years is unusual: the object is in a state of resonance with Jupiter, except that close encounters with the Earth and Mars are so frequent that the object is generally kicked to another level of resonance before a full cycle is completed. Much of this time is spent at the deepest resonance levels observed in any tests; sometimes the critical argument is librating within 10° or less about a mean of 195° to 205° . Note that during the periods of deepest resonance with Jupiter ($\sigma_{3:1}$) the absence of resonance interaction with the Earth ($\sigma_{1:4}$). Figure 24 also shows frequent jumps in semimajor axis and inclination and figure 27 the very chaotic behavior in the $p - q$ track during this period. The frequent close approaches to the Earth are at a time of low orbital inclination to the Earth's orbit— 0.5° to 0.9° —as well as proximity of the object's perihelion (0.80 to 0.98 AU) to the Earth's orbit. The inclination to the orbit of Mars is also low at this time, ranging from 1.1° to 1.4° , giving one of the highest frequencies of encounters with Mars seen in any run. Even though the resonance with Jupiter seems to resume after each close encounter this is a highly chaotic period—note the increase in $\log k_n$ in figure 26.

The object's eccentricity progressively decreases and ends crossings of the Earth's orbit by $t = 5,500$ years, giving a much more regular $p - q$ track. An uninterrupted resonance with Jupiter resumes, continuing to the end of the run; $\sigma_{3:1}$ continues to oscillate about a mean of 200° . The three close approaches to Mars towards the end of the run may involve the object's perihelion coming closer to the orbit of Mars.

5.8 Test 8

The last two tests varied the initial eccentricity. This test began with the nominal elements for Toutatis except with an eccentricity of zero to thus observe behavior near the 3:1 resonance with Jupiter apart from interaction with the inner planets. Figures 28 and 29 show the results. The eccentricity gradually increases to about 0.12; variation in semi-major axis is relatively small with the amplitude reaching a maximum of about 0.06 AU. The main cycle in the orbital elements has a period of about 800 years, with a much shorter period and shallower cycle superimposed on this. In turn these may be superimposed on a broader cycle with a period of the order of 40,000 years; this is suggested in particular by the change in eccentricity. In the graph of $\sigma_{3:1}$ versus a (fig. 29) light points indicate values before the "inversion" of $\sigma_{3:1}$ at $t = 1,050$ years and heavy points those after. This inversion and the prior behavior of $\sigma_{3:1}$ involve the poorly defined longitude of perihelion for a near-zero eccentricity. The subsequent asymptotic behavior (in contrast to a state of resonance) in fig. 29 is also seen in the mappings and integrations of Yoshikawa (1990). The plot of calculations of $\log k_n$ (fig. 30) indicates a much lower degree of chaos than tests involving encounters with the inner planets.

Evolution of low eccentricity orbits near the 3:1 resonance has been studied by several mapping techniques as well as numerical integration. In this 10,000-year run eccentricity increases from 0.0 to 0.12, the latter value apparently the maximum in a longer period cycle. This corresponds to the regular behavior seen in mappings by Wisdom for $e < 0.1$. Chaotic behavior for such objects tends to involve either much longer time spans or objects closer to the resonance position—2.501 AU versus 2.510 AU for this test object. The chaotic zone is narrower for smaller values of e . Figure 32 shows the track of this object in a and e for the run period along with (current) values for all numbered asteroids (through (5756)) with similar elements, showing the Kirkwood gap about the 3:1 resonance position at 2.501 AU. Those asteroids with values of a and e closest to those of the test object are also listed in table 5.

5.9 Test 9

The final test used nominal Toutatis elements except with an eccentricity of 0.72. This puts the test object's initial perihelion near the orbit of Venus. Several encounters with Venus do result before the eccentricity decreases enough to prevent them (fig. 33). Frequent encounters with the Earth occur in the first 5,000 years with encounters diminishing when inclination to the Earth's orbit is highest. The object is no longer Earth-crossing after about $t = 6,000$ years. Resonance with Jupiter lasts for the first 500 years and again from $t = 1,200$ to $t = 4,900$ years. The graph of $\sigma_{3:1}$ suggests that during the second period the resonance was relatively shallow and unstable, as in test 4, although in the current test encounters with the Earth occur near the time that the resonance ends. For the remainder of the run the object's semi-major axis is too far from the resonance location for resonance to continue.

6 Conclusions

These tests demonstrated the chaotic nature of Toutatis' orbit. Runs at various values of initial inclination showed the connection between inclination and frequent, chaos-producing close encounters. Several runs also suggested the additional factor of a perihelion close to the Earth's orbit (versus well inside it), although this is less critical (as well as more variable). It is quite clear that the Earth is far more important than the other inner planets in producing the chaos of Toutatis' orbit. In cases where eccentricity declines enough to end crossings of Earth's orbit, the behavior of test objects abruptly becomes less chaotic. However, frequent encounters do not necessarily eliminate resonance behavior.

Frequent close encounters certainly tend to disrupt the 3:1 mean motion resonance with Jupiter, and the resultant degree of chaos is far higher than that associated with the 3:1 resonance itself. However, there were situations in which the resonance persisted in spite of such encounters, such as test 5, or where frequent transitions to other levels of resonance occurred, as in test 7. It was clear that if Earth crossings ended with the test object near the 3:1 resonance that the resonance would take over.

A minor observation concerned the relationship of inclination, a one-dimensional value, to the orientation of the angular momentum vector, which has two degrees of freedom. The orientation of the asteroid's orbital plane varies over time, as do those for the major planets (and hence the plane of the ecliptic). Depending on what one is looking for, the orientation relative to a fixed reference (of which the invariable plane is a convenient example) may be useful, or perhaps the orientation relative to the orbital plane of a particular planet (such as when considering the pertinence of inclination to close encounters). The tracks in $p - q$ phase space relate to several different dynamical aspects.

Some observations on graphing data for studies of this type: first, the plots of close encounter maximum acceleration values versus time in Whipple and Shelus effectively present the data involved with such frequent encounters. In another regard, graphs of various elements versus time may use distinct points or a continuous line to represent the data. Plots of elements such as a and e which use distinct points have the advantage of emphasizing sudden jumps in value, whereas plots using a continuous line clarify the sequence of values when sudden increases and decreases in value both occur in a short time period. Since the critical argument σ cycles around, arbitrarily connecting all datapoints can reduce the clarity of the plot, as with such graphs in Hahn *et al.* (1991).

There have been significant developments in the past ten years in the analytic study of resonance behavior in general and of orbital evolution of Earth-approaching asteroids in particular. Some questions raised in this study concern the stability of protected resonance states, such as why some close encounters disrupt resonances and others do not. Further experiments might examine possible factors such as the depth or level of resonance or the orientation of the perturbation induced by the close encounter. The latter is one application of a more detailed analysis of individual close encounters. Other tests might omit only the influence of the Earth to compare the degree of chaos resulting from the other inner

planets. Another interesting topic is the nature of the evolution of orbital orientation, represented by the $p - q$ plots. Further projects might look at behavior of these tracks over longer periods, the tracks for objects in mean-motion resonances with inner planets, and the tracks for objects in high-inclination orbits (the highest initial inclination examined here was 5.0°), and for a means of generally characterizing the behavior of these tracks. In some of the tests conducted here, chaotic behavior or periodic resonance behavior in the orbital elements did not seem to correspond to the same type of behavior in the $p - q$ tracks.

7 References

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8 Tables

Table 1. Nominal orbital elements for (4179) Toutatis

a	2.509747568
e	0.640643763
i	0.46156101 $^\circ$
ω	275.6510933 $^\circ$
Ω	127.2323085 $^\circ$
M	226.2680053 $^\circ$
epoch	= JDE 2448400.5 = 1991 May 24.0

Elements referred to the J2000 ecliptic and equinox. From Whipple and Shelus (1993b).

Table 2. Numbered asteroids near the 3:1 resonance with eccentricity over 0.30 (in order of decreasing eccentricity)

number	and name	a (AU)	e	i ($^\circ$)
3360	1981 VA	2.46517	0.7427	21.7377
4179	Toutatis	2.50628	0.6398	0.4714
2608	Seneca	2.49008	0.5817	15.3444
1915	Quetzalcoatl	2.53640	0.5744	20.4638
887	Alinda	2.49278	0.5582	9.2512
1429	Pemba	2.55480	0.3385	7.7342
3806	1981 EW ₃₂	2.54409	0.3123	10.0552
1550	Tito	2.54532	0.3102	8.8436
1607	Mavis	2.55047	0.3078	8.5862
3628	Boznemcova	2.53841	0.3012	6.9069

Data from list provided by A. Whipple compiled from MPC listings.

Table 3. GM values and initial state vectors for planets.
Epoch = JDE 2448400.50000000

body	GM (AU ³ /d ²)
Sun	2.959122082856000D-04
Mercury	4.912547451451000D-11
Venus	7.243456209633000D-10
Earth + Moon	8.997011658557000D-10
Mars	9.549528942224000D-11
Jupiter	2.825342103446000D-07
Saturn	8.459468504831000D-08
Uranus	1.288816238138000D-08
Neptune	1.532112481284000D-08
Pluto	2.276247751864000D-12

planet	X (AU)	Y (AU)	Z (AU)
Mercury	3.159990049918950D-01	-2.120405107840322D-01	-1.460414082683861D-01
Venus	-7.167915426532021D-01	-7.057302633063772D-02	1.362122753134941D-02
Earth	-4.680340396710344D-01	-8.238237899461507D-01	-3.571930592961391D-01
Mars	-1.426634861544704D+00	7.645548559827503D-01	3.892666387829875D-01
Jupiter	-3.950797523503588D+00	3.249835032361875D+00	1.489291265522722D+00
Saturn	5.234274195772411D+00	-7.762991143709812D+00	-3.431251104924883D+00
Uranus	3.871675048144994D+00	-1.745847873096916D+01	-7.701038779594065D+00
Neptune	7.929208904105863D+00	-2.689837955915642D+01	-1.120708254148946D+01
Pluto	-1.871906856373984D+01	-2.297301075699729D+01	-1.529893220379876D+00

planet	\dot{X} (AU/d)	\dot{Y} (AU/d)	\dot{Z} (AU/d)
Mercury	1.201668310199972D-02	2.105111797503741D-02	9.997763597789049D-03
Venus	1.544599962795968D-03	-1.843047247192085D-02	-8.388767947147218D-03
Earth	1.497641658026811D-02	-7.354956710826316D-03	-3.188919315237791D-03
Mars	-6.674335168982019D-03	-9.889150370693957D-03	-4.355248018136044D-03
Jupiter	-5.163707689619151D-03	-4.872551433226798D-03	-1.962782100275474D-03
Saturn	4.442558455000389D-03	2.761907856810897D-03	9.494266931438180D-04
Uranus	3.821762318808397D-03	5.654242846859978D-04	1.936041449643723D-04
Neptune	3.005375221521944D-03	8.030911505765662D-04	2.537904434969410D-04
Pluto	2.525434889177396D-03	-2.035330237490401D-03	-1.397409820140605D-03

Data is from the JE200 ephemeris. Values of GM for planets include satellites. Cartesian heliocentric state vectors are referred to the J2000.0 equator and equinox.

Table 4. Initial orbital elements for test objects

Test	e	i ($^\circ$)	Ω ($^\circ$)	close approaches Venus-Earth-Mars
1*	0.640643763	0.46156101	127.2323085	0 18 6
2	0.640643763	0.46156101	127.2323085	0 12 0
3	0.640643763	2.0	127.2323085	3 54 5
4	0.640643763	5.0	127.2323085	0 8 2
5**	0.640643763	5.0	127.2323085	not applicable
6	0.640643763	1.5	217.0	2 15 0
7	0.640643763	1.0	37.2323085	0 55 14
8	0.0	0.46156101	127.2323085	0 0 0
9	0.72	0.46156101	127.2323085	7 38 8

* 1,500-year run; all others are 10,000 years

** inner planets turned off

Other orbital elements are the same as the nominal elements for (4179) Toutatis, given in table 1. Close approaches include those with gravitational acceleration by the planet exceeding 0.2 percent of the solar acceleration.

Table 5. Numbered asteroids with orbits similar to the test object of test 8.

number	and name	a (AU)	e	i ($^\circ$)
619	Triberga	2.51966	0.0753	13.7706
1722	Goffin	2.51370	0.0476	5.4765
2687	Tortali	2.51922	0.1225	10.0878
4002	Shinagawa	2.51675	0.0279	14.6722
4006	1972 YR	2.51721	0.1811	2.3721
5065	1990 FP ₁	2.51378	0.0485	6.5617

Data from list provided by A. Whipple compiled from MPC listings.

9 Figure captions

Fig. 1. Test 1 results, including evolution of a , e , $\sigma_{3:1}$, and $\sigma_{1:4}$. The graph of i indicates inclination with respect to the invariable plane and the orbits of Venus, the Earth and Mars by the thin solid, dashed, heavy solid, and dotted lines, respectively. For recorded close approaches to the planets Venus, the Earth, Mars and Jupiter, the remaining graphs plot the peak accelerations by the planet relative to the solar acceleration.

Fig. 2. Test 1: a vs. $\sigma_{3:1}$ (see text for explanation).

Fig. 3. Test 1: $\log k_n$ vs. $\log t$ (see text for explanation).

Fig. 4. Test 1: evolution of perihelion and aphelion.

Fig. 5. Test 1: track of $p = \sin(i) * \sin(\Omega)$ vs. $q = \sin(i) * \cos(\Omega)$ (see text for discussion).

Fig. 6. Test 2 results, in the format of fig. 1.

Fig. 7. Test 2: a vs. $\sigma_{3:1}$.

Fig. 8. Test 3 results, in the format of fig. 1.

Fig. 9. Test 3: a vs. $\sigma_{3:1}$

Fig. 10. Test 3: $\log k_n$ vs. $\log t$.

Fig. 11. Test 3: p vs. q for the test object and the planets Venus through Jupiter. Motion is clockwise for all tracks. The orientation of the invariable plane is indicated by the intersection of the axes.

Fig. 12. Test 4 results, in the format of fig. 1.

Fig. 13. Test 4: a vs. $\sigma_{3:1}$.

Fig. 14. Test 4: p vs. q (the star marks the orientation of the invariable plane).

Fig. 15. Test 4: $\log k_n$ vs. $\log t$.

Fig. 16. Test 5 results, in the format of fig. 1 except omitting graphs involving the inner planets (since this test ignores their effects). The graph of inclination indicates inclination to the invariable plane and to the orbit of Jupiter by the thin solid and heavy dotted lines, respectively.

Fig. 17. Test 5: a vs. $\sigma_{3:1}$.

Fig. 18. Test 5: p vs. q .

- Fig. 19. Test 5: $\log k_n$ vs. $\log t$.
- Fig. 20. Comparison of tests 4 and 5. Test 4 results are indicated in the graph of a by the solid line and in the graph of $\sigma_{3:1}$ by the heavy points. Results of test 5 (with the inner planets "turned off") are shown in both graphs by the light dotted lines.
- Fig. 21. Test 6 results, in the format of fig. 1.
- Fig. 22. Test 6: a vs. $\sigma_{3:1}$.
- Fig. 23. Test 6: p vs. q .
- Fig. 24. Test 7 results, in the format of fig. 1.
- Fig. 25. Test 7: a vs. $\sigma_{3:1}$.
- Fig. 26. Test 7: $\log k_n$ vs. $\log t$.
- Fig. 27. Test 7: p vs. q ; individual points are connected sequentially. Part or all of the $p - q$ tracks for the Earth, Mars, and Jupiter are also shown.
- Fig. 28. Test 8 results, in the format of fig. 16. In this low eccentricity test no close encounters with any planets occur.
- Fig. 29. Test 8: a vs. $\sigma_{3:1}$. Small and large points indicate values before and after $t = 1,050$ years, respectively.
- Fig. 30. Test 8: $\log k_n$ vs. $\log t$.
- Fig. 31. Test 8: p vs. q .
- Fig. 32. Test 8: the track of a and e for the test object (lower center) along with elements for numbered asteroids.
- Fig. 33. Test 9 results, in the format of fig. 1.
- Fig. 34. Test 9: a vs. $\sigma_{3:1}$.
- Fig. 35. Test 9: $\log k_n$ vs. $\log t$.
- Fig. 36. Test 9: p vs. q (including p vs. q for Jupiter).